حل لبعض الاسئلة من شابتر 3

ملاحظة: الحل عبارة عن صور وليس نص مكتوب. واذا لم يوجد رقم السؤال فوق الصورة، فتكون الصورة(الحل) مكملة للصورة(للحل) السابقة.

على سبيل المثال سؤال رقم 7 من سكشن 1، له صورتان.

3.1

3.1.4

The predicate for $Q(n)$ be defined as $n^2 \le 30$.

(a)

The objective is to determine which of the statements of $Q(n)$ is true for $n = 2, -2, 7,$ and -7.

Replace n with 2 in $Q(n)$.

$Q(2): 2^2 \le 30 \Rightarrow 4 \le 30.$

Thus, $Q(2)$ is **true**.

Replace n with -2 in $Q(n)$.

$Q(-2): (-2)^2 \le 30 \Rightarrow 4 \le 30.$

Thus, $Q(-2)$ is **true**.

Comment

Step 2 of 4

Replace n with 7 in $Q(n)$.

$Q(7): 7^2 \le 30 \Rightarrow 49 \not\le 30.$

Thus, $Q(7)$ is **false**.

Replace n with -7 in $Q(n)$.

$Q(-7): (-7)^2 \le 30 \Rightarrow 49 \not\le 30.$

Thus, $Q(-7)$ is **false**.

3.1

3.1.7

(a)

Consider the predicate,

Domain is all integer set $\frac{6}{d}$ is an integer.

The objective is to identify the truth set of the above predicate.

The value of d for which $\frac{6}{d}$ is an integer are $-6, -3, -2, -1, 1, 2, 3, 6$.

The domain is the set of all integers.

Therefore, the truth set is $\boxed{\{-6, -3, -2, -1, 1, 2, 3, 6\}}$.

Comment

Step 2 of 6

(b)

Consider the predicate,

Domain is all positive integers set $\frac{6}{d}$ is an integer.

The objective is to identify the truth set of the above predicate.

The value of d for which $\frac{6}{d}$ is an integer are $-6, -3, -2, -1, 1, 2, 3, 6$.

The domain is the set of all positive integers.

Therefore, the truth set is $\boxed{\{1, 2, 3, 6\}}$.

Comment

Step 3 of 6

(c)

Consider the predicate,

Domain is all real numbers set $1 \le x^2 \le 4$.

The objective is to identify the truth set of the above predicate.

3.1

The inequality can be written as,

$$1 \le x^2 \le 4$$

$$\pm 1 \le x \le \pm 2$$

That is $-1 \le x \le -2$ and $1 \le x \le 2$

The value of x for which $1 \le x^2 \le 4$ will lie between -2 and -1 inclusive together with those between 1 and 2 inclusive.

The domain is the set of all real numbers.

Therefore, the truth set is $\boxed{-2 \le x \le -1 \text{ or } 1 \le x \le 2}$.

Comment

Step 5 of 6

(d)

Consider the predicate,

Domain is set of all integer set $1 \le x^2 \le 4.$

The objective is to identify the truth set of the above predicate.

Comment

Step 6 of 6

The inequality can be written as,

$$1 \le x^2 \le 4$$

$$\pm 1 \le x \le \pm 2$$

That is $-1 \le x \le -2$ and $1 \le x \le 2$

The value of x for which $1 \le x^2 \le 4$ will lie between -2 and -1 inclusive together with those between 1 and 2 inclusive.

The domain is the set of all integers.

Therefore, the truth set is $\boxed{\{-2, -1, 1, 2\}}$.

3.1

3.1.15

(a) (i) Every rectangle is a quadrilateral.

(OR)

(ii) Any rectangle is quadrilateral.

(OR)

(iii) Some quadrilateral is a rectangular.

Comment

Step 2 of 2

(b) (i) At least one set has 16 subsets.

(OR)

(ii) We can find at least one set which has 16 subsets

3.1

3.1.16

The objective is to rewrite each of the following statements in the form of "$\forall ......x, ...$".

a.

Consider the statement as,

"All dinosaurs are extinct."

The meaning of the above statement is that each and every dinosaur is extinct.

Thus, the required statement equivalent to the given statement in the form of $\forall ......x, ...$ is,

$\boxed{\forall \text{ dinosaurs } x,\ x \text{ is extinct.}}$

Comment

Step 2 of 6

b.

Consider the statement as,

"Every real number is positive, negative, or zero."

The meaning of the above statement is that any real number has to be positive, negative or zero.

Thus, the required statement equivalent to the given statement in the form of $\forall ......x, ...$ is,

$\boxed{\forall \text{ real numvers } x,\ x \text{ is positive, negative or zero}}$.

Comment

Step 3 of 6

c.

Consider the statement as,

"No irrational numbers are integers."

The meaning of the above statement is that none of the irrational number is an integer.

Thus, the required statement equivalent to the given statement in the form of $\forall ......x, ...$ is,

$\boxed{\forall \text{ irrarional numbers } x,\ x \text{ is not an integer.}}$

3.1

3.1.18

b.

The statement is that every computer science student is an engineering student.

The above statement can be expressed as shown below,

$\forall s \in D$ if $C(s)$ then $E(s)$. (Or: $\forall s \in D$, $C(s) \rightarrow E(s)$.

In the above expression, $\forall s$ denotes for all s in the set of all students in the school.

Therefore, the expression is $\boxed{\forall s \in D \text{ if } C(s) \text{ then } E(s) \text{ Or } \forall s \in D \text{ , } C(s) \rightarrow E(s)}$.

Comment

Step 3 of 5

c.

The statement is that no computer science students are engineering students.

The above statement can be expressed as shown below,

$\forall s \in D$ such that $C(s) \sim E(s)$.

Therefore, the expression is $\boxed{\forall s \in D \text{ such that } C(s) \sim E(s)}$.

Comments (2)

Step 4 of 5

d.

The statement is that some computer science students are also math majors.

The above statement can be expressed as shown below,

$\exists s \in D$ such that $C(s) \wedge M(s)$.

Therefore, the expression is $\boxed{\exists s \in D \text{ such that } C(s) \wedge M(s)}$.

Comment

Step 5 of 5

e.

The statement is that some computer science students are engineering students and some are not.

The above statement can be expressed as shown below,

$(\exists s \in D \text{such that } C(s) \wedge E(s)) \wedge (\exists s \in D \text{such that } C(s) \wedge \sim E(s))$

Therefore, the expression is

$\boxed{(\exists s \in D \text{such that } C(s) \wedge E(s)) \wedge (\exists s \in D \text{such that } C(s) \wedge \sim E(s))}$.

3.1

3.1.22

Recall the Universal conditional statement:

" $\forall x$, if $P(x)$ then $Q(x)$"

Suppose, x is a Java program. Then, the following holds true:

$P(x)$: x is a java program.

$Q(x)$: x has at least 5 lines.

Next, apply the Universal conditional statement.

The statement in the form " $\forall$ ________ x, if____ then ______", is as follows:

" $\forall x$, if x is a Java Program, then x has at least 5 lines."

Or

In symbol form;

$\forall x(P(x) \rightarrow Q(x))$

Comment

Step 3 of 3

(b)

Consider the statement,

"Any valid argument with true premises has a true conclusion".

The objective is to write this statement in universal conditional statement form.

Suppose x is a valid argument with true premises.

Then, the following holds true:

$P(x)$: x is a valid argument with true premises.

$Q(x)$: x has true conclusion.

Next, apply Universal conditional statement.

The statement in the form " $\forall$ ________ x, if____ then ______"is as follows:

" $\forall x$, if x is a valid argument with true premises, then x has a true conclusion."

Or,

In symbol form;

$\forall x(P(x) \rightarrow Q(x))$.

3.1

3.1.23

(a)

Consider the statement,

All equilateral triangles are isosceles.

The objective is to rewrite this statement in universal conditional statement form.

The statement is rewritten as,

" $\forall x$ if x is an equilateral triangle, then x is isosceles".

And the other objective is to rewrite the given statement in not using if then form.

" $\forall$ Equilateral triangles x, x is an isosceles triangle".

Therefore, the statement can be written as $\forall$ x if x is an equilateral triangle, then x is isosceles" and " $\forall$ equilateral triangles x, x is an isosceles triangle".

Comment

Step 2 of 2

(b)

Consider the statement,

Every computer science student must take data structures course.

The objective is to rewrite this statement in universal conditional statement form.

The statement can be rewritten as,

" $\forall x$ if x is a computer science student, then x must take data structures course".

And the other objective is to rewrite the given statement in not using if then form.

" $\forall$ Computer science student x, x must take data structures course".

Therefore, the statement can be written as " $\forall$ x if x is a computer science student, then x must take data structures course" and " $\forall$ computer science student x, x must take data structures course".

3.1

3.1.25.d

(d)

The given statement is:

"The negative of any irrational number is irrational."

Here, the domain of the predicate variable is "irrational number".

Part 1: Write the given statement in the form " $\forall$ ________ x, ________".

To write the statement in this form, fill the first blank with the domain of the predicate variable "irrational number", and the second blank with the additional condition given for the predicate variable.

Therefore, the statement can be rewritten as

" $\forall$ irrational number x, the negative of x is irrational."

Comment

Step 8 of 12

Part 2: Write the given statement in the form " $\forall x$, if ________, then ________".

To write the statement in this form, fill the first blank with the domain of the predicate variable "irrational number", and the second blank with the additional condition given for the predicate variable.

Therefore, the statement can be rewritten as

" $\forall x$, if x is an irrational number, then the negative of x is irrational."

3.1

3.1.29

(a)

Consider the statement:

$\exists x$ such that $\mathrm{Rect}(x) \wedge \mathrm{Square}(x)$.

The statement can be rewritten as,

There is a shape that is both rectangle and a square.

This is a true statement since, a square is also a rectangle.

Hence, the statement is **True.**

Comment

Step 4 of 5

(b)

Consider the statement below:

$\exists x$ such that $\mathrm{Rect}(x) \wedge \square\ \mathrm{Square}(x)$

The statement can be rewritten as,

There is a shape that is a rectangle but not a square.

This is a true statement since, a rectangle with sides of length 4 and width 3 is not a square.

Hence, the statement is **True.**

Comment

Step 5 of 5

(c)

Consider the statement:

$\forall x,\ \mathrm{Square}(x) \rightarrow \mathrm{Rect}(x)$

The statement can be rewritten as,

If a shape is a square then it is also rectangle.

This is a true statement since all squares have four right angles and two sets of two sides of equal length.

Hence, the statement is **True**.

3.2

3.2.3

$\forall$ fish x, x has gills.

The formal negation is,

$\exists$ a fish x such that x does not have gills.

Therefore, the answer is $\exists$ a fish x such that x does not have gills .

Comment

Step 2 of 4

b.

The statement is,

$\forall$ computers c, c has a CPU.

The formal negation is,

$\exists$ a computer c such that c does not have a CPU.

Therefore, the answer is $\exists$ a computer c such that c does not have a CPU .

Comment

Step 3 of 4

c.

The statement is,

$\exists$ a movie m such that m is over 6 hours long.

The formal negation is,

$\forall$ movies m, m is less than or equal to 6 hours long.

Therefore, the answer is $\forall$ movies m, m is less than or equal to 6 hours long .

Comment

Step 4 of 4

d.

The statement is,

$\exists$ a band b such that b has won at least 10 Germany awards.

The formal negation is,

$\forall$ band b, b has won less than 10 Germany awards

Therefore, the answer is $\forall$ band b, b has won less than 10 Germany awards .

3.2

3.2.7

The statement is "There are no orders from store *A* for item *B*".

If the statement is in the form where the word "Some are" or "At least" is used, then it is called existential statement.

But here the statement is for all orders not for some orders.

Thus, the statement "There are no orders from store *A* for item *B*" is not existential.

Comment

Step 2 of 2

Now, the informal negation of the statement "There are no orders from store *A* for item *B*" can be written as:

"There is at least one order from store *A* for item *B*".

And,

Formal version of the statement "There are no orders from store *A* for item *B*" can be written as:

" $\forall$ order *x*, if *x* is from store *A*, then *x* is not for item *B*".

3.2.10

$\exists$ computer program *P* such that *P* compiles without error messages and *P* is incorrect

3.2

3.2.14

The statement is shown below,

"For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 = x_2$".

The proposed negation is,

"For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 \neq x_2$".

The objective is to determine if the proposed negation is correct.

The proposed negation is not correct because for the statement "For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 = x_2$", to be false there must be at least one pair of real numbers such that $x_1^2 = x_2^2$ then $x_1 \neq x_2$.

The proposed negation "For all real numbers x_1 and x_2, if $x_1^2 = x_2^2$ then $x_1 \neq x_2$" means that given any two real numbers, $x_1^2 = x_2^2$ then $x_1 \neq x_2$.

The truth of this statement implies the truth of the negation, but the negation can be true without having its statement true.

Therefore, the correct negation is "There exists two real numbers x_1 and x_2 such that $x_1^2 = x_2^2$ and $x_1 \neq x_2$".

3.2.23

<u>Negation:</u>

There is a function such that function is differentiable but the function is not continuous.

Or

There is at least one differentiable function but is not continuous.

3.2

3.2.24.a

(a)

Consider the statements,

'The total children in Tom's family are female'.

'The total females in Tom's family are children'.

The objective is to rewrite the above statements in if-then form.

The first statement in conditional statement form:

If a person is a child in Tom's family, then the person is female.

The second statement in conditional statement form:

If a person is female in Tom's family, then the person is child.

Comment

Step 2 of 4

Converse of the statement,

If $p \to q$ is the statement, then $q \to p$ is the converse of the statement.

Next, objective is to write the logical relationship between the above two statements.

The logical relationship between the above statements as,

The second statement is the converse of the first statement.

That is 'The total females in Tom's family are children' is the converse of 'The total children in Tom's family are female'.

3.2

3.2.24.b

(b)

Consider the statements,

'The total integers greater than 5 and end in 1, 3, 7, or 9 are prime numbers'.

'The total integers that are greater than 5 and are prime numbers end in 1, 3, 7, or 9'.

The objective is to rewrite the above statements in if-then form.

The first statement in conditional statement form:

If an integer is greater than 5, then the integer ending in 1, 3, 7, or 9 are prime.

If an integer is a prime which is greater than 5, then the integer ends in 1, 3, 7, or 9.

Comment

Step 4 of 4

The objective is to write the logical relationship between the above two statements.

The logical relationship between the above statements as,

The second statement is the converse of the first statement.

That is 'The total integers greater than 5 and end in 1, 3, 7, and 9 are prime numbers' is the converse of 'The total integers that are greater than 5 and are prime numbers end in 1, 3, 7, or 9'.

3.2

3.2.27

Consider the statement,

If $\frac{6}{d}$ is an integer, then $d = 3$ for all integers $d.$

The objective is to write the converse, inverse, and contrapositive for above statement.

Identify the best among the statement, its converse, its inverse, and its contrapositive are true, and which are false.

If the statement is false, give the counterexample.

Comment

Step 2 of 5

The statement can be written as,

$p : \frac{6}{d}$ and $q : d = 3.$

The statement is 'If $\frac{6}{d}$ is an integer, then $d = 3$ for all integers $d.$'

Provided statement is **false**.

Hence, $p \to q$

Counterexample:

Let $d = 2$.

$$\begin{aligned} \frac{6}{d} &= \frac{6}{2} \\ &= 3 \\ &= \textbf{Integer} \end{aligned}$$

Here, 3 is an also integer but $d \neq 3$.

Hence, the statement $p \to q$ is false.

3.2

Converse of the statement:

If q, then p.

Converse of the above statement is,

If $d = 3$, then $\frac{6}{d}$ is an integer for all integers d.

This converse is true.

$\frac{6}{d} = \frac{6}{3} = 2$

Here 2 is also an integer.

Comment

Step **4** of 5

Inverse of the statement:

If $\sim p$, then $\sim q$.

Inverse of the above statement is,

If $\frac{6}{d}$ is not an integer, then $d \neq 3$ for all integers d.

This inverse is **true**.

$\frac{6}{d} = \frac{6}{4} = \frac{3}{2}$ is not an integer.

Here $d = 4 \neq 3$.

Comment

Step **5** of 5

Contrapositive of the statement:

If $\sim q$, then $\sim p$.

Contrapositive of the above statement is,

If $d \neq 3$, then $\frac{6}{d}$ is not an integer for all integers d.

This contrapositive is **false**.

If $d = 2 \neq 3$, then $\frac{6}{d} = \frac{6}{2} = 3$ is an integer.

3.2

3.2.36

(a)

Consider $P(x)$ is a predicate and the domain of x is the set of all real numbers.

And the statements as follows,

R be " $\forall x \in Z, P(x)$"

S be " $\forall x \in Q, P(x)$"

T Be " $\forall x \in R, P(x)$"

The objective is to find a definition for $P(x)$ so that R is true and both S and T are false and not using $x \in Z$.

Comment

Step 2 of 5

Suppose $P(x)$ be $5x \neq 1$.

Then the negation is $\sim P(x)$ be $5x = 1$.

Since $\forall x \in Z, 5x \neq 1$

Therefore, " $\forall x \in Z, P(x)$" is true.

Hence, R is true.

Comment

Step 3 of 5

Since $5\left(\frac{1}{5}\right) = 1$, for $x = \frac{1}{5} \in Q$, $5x = 1$.

$\exists x = \frac{1}{5} \in Q$, $x \in \sim P(x)$.

Therefore, " $\forall x \in Q, P(x)$" is false.

Hence, S is false.

3.2

Since $5\left(\frac{1}{5}\right)=1$, for $x=\frac{1}{5}\in R$, $5x=1$.

$\exists x=\frac{1}{5}\in R$, $x\in\sim P(x)$.

Therefore, " $\forall x\in R, P(x)$ " is false.

Hence, T is false.

Therefore, the definition of the predicate $P(x)$ is,

$\boxed{P(x) \text{ be } 5x\neq 1}$.

|

(b)

The objective is to find a definition for $P(x)$ so that both R and S are true and T is false and not using $x\in Q$.

Suppose $P(x)$ be $5x\neq\sqrt{2}$.

Then the negation is $\sim P(x)$ be $5x=\sqrt{2}$.

Since $\forall x\in Z, 5x\neq\sqrt{2}$.

Therefore, " $\forall x\in Z, P(x)$ " is true.

Hence, R is true.

Since $\forall x\in Q, 5x\neq\sqrt{2}$.

Therefore, " $\forall x\in Q, P(x)$ " is true.

Hence, S is true.

Comment

Step 5 of 5

Since $5\left(\frac{\sqrt{2}}{5}\right)=\sqrt{2}$, for $x=\frac{\sqrt{2}}{5}\in R$, $5x=\sqrt{2}$.

$\exists x=\frac{\sqrt{2}}{5}\in R$, $x\in\sim P(x)$.

Therefore, " $\forall x\in R, P(x)$ " is false.

Hence, T is false.

$\boxed{P(x) \text{ be } 5x\neq\sqrt{2}}$.

3.2

3.2.43

It is not the case that, if a number is divisible by 4, then that number is divisible by 8

In other words, there is a number that is divisible by 4 and is not divisible by 8

3.2.44

The given statement is:

"Having a large income is not a necessary condition for a person to be happy."

Now, " $\forall x, r(x)$ is a necessary condition for $s(x)$" means " $\forall x$, if □ $r(x)$ then □ $s(x)$" or equivalently, " $\forall x$, if $s(x)$ then $r(x)$".

The given statement can be rewritten as " $\forall$ person, a person having a large income is not a necessary condition for a person to be happy".

Hence in the given statement, $r(x)$ is "a person having a large income", and $s(x)$ is "a person to be happy".

Comment

Step 2 of 2

Now, " $\forall x, r(x)$ is a necessary condition for $s(x)$" means " $\forall x$, if $s(x)$ then $r(x)$".

Therefore, " $\forall x, r(x)$ is not a necessary condition for $s(x)$" means " $\exists x$ such that $s(x) \wedge$ □ $r(x)$".

Replace *x*, $r(x)$ and $s(x)$ with the appropriate wordings to understand the meaning of the given statement.

" $\exists$ person such that the person is happy and the person is not having a large income".

Therefore, the statement can be rewritten as

"There is a person who is happy and does not have a large income".

3.2

3.2.45

It is not the case that, if a person has a large income, then that person is happy

In other words, there is a person who has a large income and is not happy

3.2.46

It is not the case that, if a function is polynomial, then the function has a real root

In other words, there is a function that is polynomial and doesn't have a real root

3.3

3.3.2

Let $G(x, y)$ be $x^2 > y$

(a) $G(2,3)$ is true because $2^2 = 4 > 3$

Comment

Step **2** of 4

(b) $G(1,1)$ is false because $1^2 = 1 \not> 1$

Comment

Step **3** of 4

(c) $G\left(\frac{1}{2}, \frac{1}{2}\right)$ is false because $\left(\frac{1}{2}\right)^2 = \frac{1}{4} \not> \frac{1}{2}$

Comment

Step **4** of 4

(d) $G(-2,2)$ is true because $(-2)^2 = 4 > 2$

3.3

3.3.6

There are four squares *e*, *g*, *h* and *j*. The following table shows that for each of these squares, a circle of different color can be found such that it is above the square.

Given x =	Choose y =	Check that x and y have different colors	Check that y is above x
e	*a*, *b* or *c*	Yes	Yes
g	*a* or *c*	Yes	Yes
h	*a* or *c*	Yes	Yes
j	*b*	Yes	Yes

Since for every square *e*, *g*, *h* and *j*, we are able to find a circle such that the square and the circle have different colors, and the circle is above the square, the given statement is true.

3.3

3.3.11

a.

Consider the statement as,

" $\exists s \in S$ such that $V(s, \text{Casablanca})$".

Represent the key symbols as:

$\exists s \in S$: There is at least one student.

$V(s, \text{Casablanca})$: Student s has seen the movie Casablanca.

So, the statement can be written as:

There is a student at school who has seen the movie Casablanca.

Comment

Step 2 of 6

b.

Consider the statement as,

" $\forall s \in S$ such that $V(s, \text{Star Wars})$".

Represent the key symbols as:

$\forall s \in S$: All students at School.

$V(s, \text{Star Wars})$: Student s has seen the movie Star Wars.

So, the statement can be written as:

All students at school have seen the movie Star Wars.

Comment

Step 3 of 6

c.

Consider the statement as,

" $\forall s \in S, \exists m \in M$ such that $V(s, m)$".

Represent the key symbols as:

$\forall s \in S$: All students at School.

$\exists m \in M$: At least one movie.

$V(s, m)$: Some student s has seen some movie m.

So, the statement can be written as:

Every student at the school has seen at least one movie.

3.3

e.

Consider the statement as,

" $\exists s \in S, \exists t \in S$, and $\exists m \in M$ such that $s \neq t$ and $V(s,m) \wedge V(t,m)$".

Represent the key symbols as:

$\exists s \in S$: Some student s at School.

$\exists t \in S$: Some student t at School.

$\exists m \in M$: At least one movie.

$s \neq t$: The students s and t are different.

$V(s,m)$: Student s has seen some movie m.

$\wedge$: and

$V(t,m)$: Student t has seen some movie m.

So, the statement can be written as:

There are two different students at school that have seen the same movie.

Comment

Step 6 of 6

f.

Consider the statement as,

" $\exists s \in S$ and $\exists t \in S$ such that $s \neq t$ and $\forall m \in M, V(s,m) \rightarrow V(t,m)$".

Represent the key symbols as:

$\exists s \in S$: Some student s at School.

$\exists t \in S$: Some student t at School.

$s \neq t$: The students s and t are different.

$\exists m \in M$: At least one movie.

$V(s,m)$: Student s has seen some movie m.

$\rightarrow$: implies

$V(t,m)$: Student t has seen some movie m.

So, the statement can be written as:

There are two students such that if one student has seen a movie, then the other student also has seen the movie.

3.3

3.3.13

(a) Statement

For every color, there is an animal of that color

There are animals of every color

Comment

Step 2 of 2

(b) Negation

$\exists$ a color *C* such that $\forall$ animals *A*, *A* is not colored *C*

For some color, there is no animal of that color

3.3.18

(a) Statement

For every real number *x*, there is a real number *y* such that $x + y = 0$

Given any real number *x*, there exists a real number *y* such that $x + y = 0$

Given any real number, we can find another real number (possibly the same one) such that the sum of the given number plus the other number equals 0

Every real number can be added to some other real number (possibly itself) to obtain 0

Comment

Step 2 of 2

(b) Negation

$\exists$ a real number *x* such that $\forall$ real numbers *y*, $x + y \neq 0$

There is a real number *x* with the property $x + y \neq 0$ for any real number *y*

Some real number has the property that its sum with any other real number is non-zero

3.3

3.3.22

(a)

Consider the statement,

$\forall$ real numbers x, $\exists$ a real number y such that $x+y=0$.

The objective is to rewrite the statement without using variables or the symbols.

And identify whether this statement is true or false.

Comment

Step 2 of 4

The statement can be rewritten as,

For any real number, you can find a real number so that the sum of the two numbers is 0.

The statement is true because every real number has its own additive inverse.

Example:

$2-2=0$

Therefore, the statement is true.

Comment

Step 3 of 4

(b)

Consider the statement,

$\exists$ a real number y such that $\forall$ real numbers x, $x+y=0$.

The objective is to rewrite the statement without using variables or the symbols.

And identify whether this statement is true or false.

Comment

Step 4 of 4

The statement can be rewritten as,

There is a real number whose sum with any real number is zero.

The statement is false because no numbers will work for all the numbers.

Adding of two different numbers does not occur zero.

Therefore, the statement is false.

3.3

3.3.27

(a)

The objective is to determine whether the statement is true or false.

This statement is a true statement, because there is a circle *b* lies above the square *e* and they are different colors.

Hence, the above statement is true.

Comment

Step 3 of 3

(b)

The objective is to write a negation for the mentioned statement.

The negation of the above statement is as follows:

For all circles x and for all squares y, x is not above y or x and y are of the same colors.

3.3.31

Consider the statement

Everybody is older than somebody

This can be rewritten as

$\forall$ people *x*, $\exists$ a person *y* such that *x* is older than *y*

3.3

3.3.38

The statement is: Every action has an equal and opposite reaction.

(a)

The objective is to write the statement using quantifiers and variables.

Define D to be the set of actions and reactions.

Define $P(x,y) = $ "x **and** y **are equal and opposite**" .

The statement can be written as:

$\forall$ **action** x, $\exists$ **a reaction** y, **such that** x **and** y **are equal and opposite.**

That is, $\forall x \in D, \exists y \in D : P(x,y)$.

Comment

Step 2 of 2

(b)

The objective is to write the negation for the statement.

Apply negation to the statement in part **(a).**

$$\begin{aligned} &\sim\left(\forall x \in D, \exists y \in D : P(x,y)\right) \\ &\qquad \equiv \exists x \in D, \sim\left(\exists y \in D : P(x,y)\right) \quad \textbf{by negating } \forall x \\ &\qquad \equiv \exists x \in D, \forall y \in D : \sim P(x,y) \qquad \textbf{by negating } \exists y \end{aligned}$$

In words, the statement can be written as:

There is some action that is not equal and opposite to any reaction.

3.3

3.3.41

b)

TRUE. Rewrite the statement as shown below:

"For all integers x, there exist an integer y, such that first integer is one more than second one."

Consider any arbitrary integer x, then for every integer x, there is also an integer y such that $y = x - 1$.

$y + 1 = (x - 1) + 1 = x$

Thus, the statement is TRUE.

Comment

Step 3 of 8

c)

FALSE. Rewrite the statement as shown below:

"There exist real number x, such that for all real y, first number is one more than second one."

Consider $y = 1$,

$$\begin{aligned} x &= y + 1 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

Consider $y = 2$,

$$\begin{aligned} x &= y + 1 \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

For any real number y, there is no fixed x such that $x = y + 1$.

Thus, the statement is FALSE.

Comment

Step 4 of 8

d)

TRUE. Rewrite the statement as shown below:

"For all positive real x, there exist a positive real y, such that their product is unit."

For any given positive real number x, we can consider $y = \frac{1}{x}$, which is also positive real number and $xy = 1$.

Thus, the statement is TRUE.

3.3

f)

FALSE. Rewrite the statement as shown below:

"For all positive integer x and all positive integer y ,there exist a $+ve$ integer z , such that third one is the difference of first and second."

Consider $x = 1, y = 2$.

$$\begin{aligned} z &= x - y \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

For integer x which is less than the integer y, $z = x - y$ is always negative.

Thus, the statement is FALSE.

Comment

Step 7 of 8

g)

TRUE. Rewrite the statement as shown below:

"For all integer x and all integer y, there exist an integer z, such that third one is the difference of first and second."

For any integer x and integer y, $z = x - y$ is always an integer.

Thus, the statement is TRUE.

Comment

Step 8 of 8

h)

TRUE. Rewrite the statement as shown below:

"For all positive real numbers v, there exist positive real u such that $uv < v$, In fact for $0 < u < 1$.

$$uv - v = v(u-1) < 0$$

$$\Rightarrow uv < v$$

Thus, the statement is TRUE.

3.3

3.3.46

(a)

The given statement "There is a triangle x such that for all squares y, x is above y".

The statement is true because of the following reason.

In the Tarski world of figure 3.3.1 given, there are two triangles namely a in the first row and c in the second row.

If someone gives you a triangle a or c then it can be observed that they are above all the three squares e, h and j in third, fourth and fifth rows respectively. So for all the squares the property that one triangle is above all the squares is true.

Comment

Step 2 of 3

(b)

The statement can be written using formal logical notation using the example 3.3.10 as below.

Statement: $\exists x(\mathbf{Triangle}(x) \wedge \forall y \mathbf{Square}(y) \rightarrow \mathbf{Above}(x, y))$

Comment

Step 3 of 3

(c)

The negation of the statement in (b) is given below.

Statement: $\exists x(\mathbf{Triangle}(x) \wedge \forall y \mathbf{Square}(y) \rightarrow \mathbf{Above}(x, y))$

Negation: $\sim(\exists x(\mathbf{Triangle}(x) \wedge \forall y \mathbf{Square}(y) \rightarrow \mathbf{Above}(x, y)))$

$\equiv \forall x \sim (\mathbf{Triangle}(x) \wedge \forall y \mathbf{Square}(y) \rightarrow \mathbf{Above}(x, y))$ (By the law of negating a $\exists$ statement)

$\equiv \forall x(\sim \mathbf{Triangle}(x) \vee \sim (\forall y \mathbf{Square}(y) \rightarrow \mathbf{Above}(x, y)))$ (By Demorgan's Law)

$\equiv \forall x(\sim \mathbf{Triangle}(x) \vee (\exists y \sim (\mathbf{Square}(y) \rightarrow \mathbf{Above}(x, y)))$ (By the law of negating $\forall$ statement)

$\equiv \forall x(\sim \mathbf{Triangle}(x) \vee (\exists y(\mathbf{Square}(y) \wedge \sim \mathbf{Above}(x, y)))$ (By the law of negating if-then statement)

3.3

3.3.48

The statement is "For all circle *x*, there is a square *y* such that *y* is to the right of *x*".

(a)

The statement says that you must find all the square that is right to the circle.

But in Tarski 3.3.10,

For all the circles $b, d, f,$ and i, there is a square to the right of these circles.

But there is no square to the right of the circle *k*.

Thus, the statement is false.

Comment

Step 2 of 3

(b)

Let Circle (x), Square (y) and RightOf (y, x) mean "*x* is circle", "*y* is square", and "*y* is to the right of *x*".

Thus, the statement using formal logical notation is:

$\forall x$ (Circle $(x) \rightarrow (\exists y$ (Square $(y) \wedge$ RightOf $(y, x))))$

Comment

Step 3 of 3

(c)

Now, the formal negation of the statement can be written as:

$\neg(\forall x$ (Circle $(x) \rightarrow (\exists y$ (Square $(y) \wedge$ RightOf $(y, x)))))$

$\equiv \exists x \neg$(Circle $(x) \rightarrow (\exists y$ (Square $(y) \wedge$ RightOf $(y, x))))$

(by the law for negating a $\forall$ statement)

$\equiv \exists x$(Circle $(x) \wedge \neg(\exists y$ (Square $(y) \wedge$ RightOf $(y, x))))$

(by the law for negating an if-then statement)

$\equiv \exists x$(Circle $(x) \wedge \forall y(\neg$(Square $(y) \wedge$ RightOf $(y, x))))$

(by the law for negating a $\exists$ statement)

$\equiv \exists x$(Circle $(x) \wedge \forall y(\neg$(Square $(y) \vee \neg$ RightOf $(y, x))))$

(by De Morgan's law)